

## Flux stabilization in 6 dimensions: D-terms and loop corrections

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**ABSTRACT:** We analyse  $D$ -terms induced by gauge theory fluxes in the context of 6-dimensional supergravity models. On the one hand, this is arguably the simplest concrete setting in which the controversial idea of ‘ $D$ -term uplifts’ can be investigated. On the other hand, it is a very plausible intermediate step on the way from a 10d string theory model to 4d phenomenology. Our specific results include the flux-induced one-loop correction to the scalar potential coming from charged hypermultiplets. Furthermore, we comment on the interplay of gauge theory fluxes and gaugino condensation in the present context, demonstrate explicitly how the  $D$ -term arises from the gauging of one of the compactification moduli, and briefly discuss further ingredients that may be required for the construction of a phenomenologically viable model. In particular, we show how the 6d dilaton and volume moduli can be simultaneously stabilized, in the spirit of KKLT, by the combination of an R symmetry twist, a gaugino condensate, and a flux-induced  $D$ -term.

**KEYWORDS:** Supergravity Models, Field Theories in Higher Dimensions, Flux compactifications.

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**1. Introduction**

Fluxes are an essential ingredient in the compactification of string-theoretic and other higher-dimensional models (see [1] for a recent review). In the present paper, we analyse the effects that  $D$ -terms induced by gauge theory fluxes can have in the context of moduli stabilization of 6d supergravity compactified to 4 dimensions. Although, at a technical level, the paper is entirely field-theoretic and relies only on the familiar 6d supergravity lagrangian of [2], our motivation is largely string-theoretic, as we now explain:

One of the perceived problems of the KKLT construction [3] of metastable de-Sitter vacua of type IIB supergravity is the presence of  $\overline{D3}$ -branes (‘anti-D3-branes’), which break supersymmetry explicitly. Before supersymmetry breaking, the model is characterized by the superpotential

$$W = W_0 + Ae^{-aT} , \tag{1.1}$$

where  $T$  is a chiral superfield with no-scale Kähler potential. The AdS vacuum of this model is then ‘uplifted’ by adding  $\overline{D3}$ -branes in a strongly warped region. Since no  $\mathcal{N} = 1$  supergravity description of this construction has so far been derived from first principles, their effect is usually incorporated by adding an uplifting term [3, 4]

$$V_{\overline{D3}} \sim \frac{1}{(T + \overline{T})^2} \tag{1.2}$$

directly to the scalar potential (i.e. without specifying the modified supergravity model).<sup>1</sup>

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<sup>1</sup>It has, however, been argued that a phenomenologically motivated description in terms of non-linearly realized supersymmetry is sufficient for most practical purposes [5]. This is consistent with the observation that, when modelling the  $\overline{D3}$  brane uplift by  $F$ -term breaking, the phenomenology turns out to be independent of the detailed dynamics of this SUSY breaking sector (unless extra fields violate the underlying sequestering assumption) [6].

Following Burgess, Kallosh and Quevedo [7], one can attempt to avoid these difficulties by introducing, instead of  $\overline{\text{D3}}$ -branes, supersymmetry-breaking two-form flux on the worldvolume of D7-branes. This has a well-known  $\mathcal{N} = 1$  supergravity description in terms of a supersymmetry-breaking  $D$ -term potential (see e.g. [8–11])

$$V_D \sim \frac{D^2}{T + \overline{T}} \sim \frac{1}{(T + \overline{T})^3}. \tag{1.3}$$

As emphasized by a number of authors [12, 5, 13, 14], there are, however, two fundamental problems with this proposal: one related to the intimate connection between  $F$ - and  $D$ -terms, the other to the gauge invariance of the superpotential. The second problem becomes apparent if one recalls that  $D$ -terms originate from the gauging of an isometry of the scalar manifold of the supergravity model [15]. In the present case, the relevant symmetry is the shift symmetry acting on the imaginary part of  $T$  (see [9] for a detailed discussion). However, the superpotential of eq. (1.1) is not invariant under a shift in  $\text{Im}(T)$ , rendering the whole construction inconsistent. To be more precise, while a transformation of  $W$  by a complex phase could be tolerated (being equivalent to a Kähler-Weyl transformation), the presence of the constant  $W_0$  induces a more complicated behaviour and thus an actual inconsistency. Thus, there is a clash between three ingredients: the 3-form-flux-induced constant  $W_0$ , the gaugino condensate inducing the  $\exp(-aT)$  contribution [16], and the gauging of the shift symmetry in  $\text{Im}(T)$ . Any two of these three ingredients may be able to coexist in a consistent model.

The above clash can obviously be avoided if light fields other than  $T$  are present and the coefficient  $A$  of the exponential term in eq. (1.1) depends on them in such a way as to render  $W$  gauge invariant. Indeed, this is well-known to occur in 4d supersymmetric gauge theory [17] (see [12, 18] for a discussion in the present context). It has very recently been demonstrated [11] that, as expected, consistent type IIB compactifications avoid any potential inconsistency between 2-form flux and gaugino condensation by precisely this mechanism.<sup>2</sup>

However, this resolution of the gauge-invariance problem has serious implications for the whole stabilization/uplifting proposal. To see this, let  $A = A(\Phi_1, \dots, \Phi_n)$  and formulate the gauge-invariance requirement for  $W$  as

$$(X^{\Phi_i} \partial_{\Phi_i} + X^T \partial_T)W = 0, \tag{1.4}$$

where  $X$  is the Killing vector of the isometry to be gauged. We can now re-parameterize the scalar manifold as follows: Choose some complex  $n$ -dimensional submanifold transverse to  $X$ , parameterize it arbitrarily by  $n$  variables, and finally parameterize the motion of this manifold along  $X$  by one last variable  $z$ . Clearly, in this parameterization  $X^z$  is the only non-zero component of the Killing vector and  $W$  is independent of the  $z$  superfield. Thus, in the original AdS vacuum,  $D_z W = K_z W = 0$  and hence  $K_z = \partial K / \partial z = 0$ . This implies

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<sup>2</sup>For a discussion of the related clash between the gauging of isometries and superpotential corrections by instantons see [19].

that the  $D$ -term arising after the gauging of the  $X$ -isometry,  $D = iK_z X^z$ , automatically vanishes at the point of the AdS vacuum.<sup>3</sup>

Does this mean that the AdS vacuum of KKL<sup>T</sup> can not be uplifted by a small correction related to the gauging of an isometry? We would like to argue the opposite as follows. The  $D$ -term uplift has to be understood as a twofold modification of the model in which the AdS vacuum occurs: One ingredient is the inclusion of extra light fields ( $A \rightarrow A(\Phi)$  in the simplest case), the other is the gauging of an isometry. Without loss of generality, we can assume

$$W = W_0 + Ae^{-a(\Phi+T)}, \tag{1.5}$$

with  $A$  an appropriately redefined constant. Furthermore, we can make the ansatz

$$K = -3\ln(T + \bar{T}) + g(\Phi + \bar{\Phi}) \tag{1.6}$$

for the Kähler potential, which allows us to gauge the isometry  $T \rightarrow T + i\delta$ ,  $\Phi \rightarrow \Phi - i\delta$ . Allowing ourselves to choose an arbitrary functional form for  $g$  it is clearly possible to arrange for the scalar potential

$$V = e^K \left[ K_{\Phi\bar{\Phi}}^{-1} |D_{\Phi}W|^2 + K_{T\bar{T}}^{-1} |D_TW|^2 - 3|W|^2 \right] + \frac{1}{2}(\text{Re}h)^{-1}D^2 \tag{1.7}$$

to have a minimum near the original AdS vacuum with small  $F$  terms and a sizeable  $D$ -term  $D$ . (Here  $h$  is the gauge-kinetic function.) This might then be viewed as a  $D$ -term uplift of the original SUSY AdS vacuum.

Moreover, the following can be viewed as a limiting case of the above proposal: Leave the  $T$  sector of the model, responsible for the SUSY AdS vacuum, completely unchanged (avoiding in particular any attempt to gauge the shift in  $\text{Im}T$ ). Instead, add an extra superfield  $\Phi$  and gauge it in such a way that, in the vacuum, the  $D$ -term dominates over the  $F$ -term. One might consider such an approach as a  $D$ -term analogue<sup>4</sup> of uplifts by nonlinearly realized SUSY [5], by  $F$ -terms in the strongly warped region [6], or by dynamical SUSY breaking [24] (see also [25, 26]). It is not known whether this or the previously outlined variant of a  $D$ -term uplift have a string-theoretic realization, but there appear to be no fundamental inconsistencies.

In our following investigation of 6d supergravity with 2-form-flux [27–29] (see also [30] for recent related work), we will not be able to realize one of these conceivable scenarios to full satisfaction. However, we will develop a number of ingredients that may be useful in this pursuit in the future.

We begin in section 2 by analysing in detail a simple  $T^2/Z_2$  model (easily generalizable to  $T^2/Z_n$ ) in which two modulus superfields  $S$  and  $T$  encode (different combinations of) the dilaton and the compactification volume. We calculate the scalar potential arising in

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<sup>3</sup>This last statement is a simplified rendition of the above-mentioned  $F$ -term/ $D$ -term problem. It can, in principle, be avoided by allowing for a Fayet-Iliopoulos term (an additive constant contribution to  $D$  which is not proportional to  $K_z$ ). However, such effects do not arise in the present context of 2-form-flux-induced  $D$ -terms. Furthermore, the above does clearly not represent an objection to the  $D$ -term uplift [20] of non-SUSY AdS vacua [21] arising from the interplay of  $\alpha'$  corrections [22] and Kähler corrections.

<sup>4</sup>Phenomenological constraints on non-sequestered  $D$ -term uplifts were discussed in [23].

the presence of 2-form-flux in two ways — by integrating the  $F_{56}^2$  term over the compact space and by finding the  $D$ -term that arises from the gauge transformation of  $T$ . Since the superfield  $S$ , which governs all gauge-kinetic functions, does not transform, no gauge invariance problem arises in the presence of gaugino condensation.<sup>5</sup>

We continue in section 3 by calculating the one-loop correction to the scalar potential that arises if hypermultiplets charged under the fluxed U(1) are present. Its parametrical behaviour is that of a usual Casimir energy, i.e.  $\sim 1/R^4$  in the Brans-Dicke frame (the frame where the coefficient of the 4d Einstein-Hilbert term is proportional to the torus volume  $R^2$ ). Due to the quantized coefficient of this loop correction, it is potentially more important than Casimir energies induced by other (weak) SUSY breaking effects.

One such SUSY breaking effect, which we discuss in section 4, is 6d Scherk-Schwarz breaking. In close analogy to the more familiar 5d case, it is implemented using an  $SU(2)_R$ -symmetry twist and can be viewed, from the 4d perspective, as the introduction of a constant superpotential  $W_0$ . We also comment on the (im-)possibility of this type of SUSY breaking on  $Z_n$  orbifolds with various  $n$  and on other mechanisms for the generation of a non-zero superpotential.

In section 5 we discuss options for moduli stabilization using the various ingredients analysed above. Working on a  $T^2/Z_2$  orbifold and ignoring, for simplicity, the shape modulus of the torus, one still has to deal with the stabilization of the superfields  $S$  and  $T$  simultaneously. At fixed  $T$ , the modulus  $S$  is stabilized à la KKLT by the interplay of  $W_0$  and gaugino condensate. The depth of the resulting SUSY AdS vacuum depends on  $T$ , driving  $\text{Re}T$  to small values. This is balanced by the  $T$  dependence of the flux-induced  $D$ -term, leading to a stable non-SUSY AdS vacuum. Thus, while the 2-form flux does not provide the desired uplift, it plays an essential role in the simultaneous stabilization of two moduli. Unfortunately, the loop correction has the same  $T$  dependence as the flux term (being suppressed by an extra power of  $\text{Re}S$ ) so that an uplift using the former is impossible (at least within our step-by-step approximate analysis). However, we consider the possibility of a simultaneous stabilization of two moduli by the interplay of  $W_0$ , gaugino condensate and  $D$ -term an interesting and positive result. The required uplift can, in the present context, be provided by  $F$ -terms of the  $\mathcal{N} = 1$  sectors localized at the orbifold fixed points.

Our conclusions are given in section 6 and some technical details of the loop calculation are relegated to the appendix.

## 2. A six-dimensional model

We work with the following bosonic action for supergravity coupled to gauge theory in six dimensions [2, 27]:<sup>6</sup>

$$\sqrt{-g_6}^{-1} \mathcal{L} = -\frac{1}{2} \mathcal{R}_6 - \frac{1}{2} \partial_M \phi \partial^M \phi - \frac{1}{24} e^{2\phi} H_{MNP} H^{MNP} - \frac{1}{4} e^\phi F_{MN} F^{MN}. \quad (2.1)$$

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<sup>5</sup>A related situation occurring in the presence of both flux and gaugino condensation on the *same* D7-brane-stack is discussed in the appendix of [11].

<sup>6</sup>We use the conventions of appendix B of [31]. Note that our action contains a tensor multiplet besides the supergravity and the vector multiplet. If one wants to work with a Lorentz invariant action this enlargement of the minimal setup is unavoidable [32].

The field strength  $H$  is defined as

$$H_{MNP} = \partial_M B_{NP} + F_{MNA}P + \text{cyclic permutations} = (dB + F \wedge A)_{MNP} \quad (2.2)$$

and the above action is invariant under the gauge transformations

$$\delta A = d\Lambda, \quad \delta B = -\Lambda F + dC. \quad (2.3)$$

The extra symmetry related to the Kalb-Ramond  $B$ -field and implemented by the 1-form  $C$  will be crucial in the presence of fluxes for  $F$ . The metric of the six-dimensional spacetime  $R^4 \times T^2$  is taken to be

$$(g_6)_{MN} = \begin{pmatrix} r^{-2}(g_4)_{\mu\nu} & 0 \\ 0 & r^2(g_2)_{mn} \end{pmatrix}, \quad (2.4)$$

with  $\mu, \nu = 0 \dots 3$  and  $m, n = 5 \dots 6$ . The  $r^2$  in front of  $(g_2)_{mn}$  controls the size of the extra dimensions in a convenient fashion, whereas the  $r^{-2}$  in front of  $(g_4)_{\mu\nu}$  acts as an automatic Weyl rescaling to ensure that the Einstein-Hilbert term in 4D is canonical. The metric of the internal space is

$$(g_2)_{mn} = \frac{1}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 \end{pmatrix}, \quad (2.5)$$

with the modulus  $\tau \equiv \tau_2 + i\tau_1$  controlling the shape of the torus. The domain of  $x_5$  and  $x_6$  is taken to be a square of unit length, so that  $\int \sqrt{g_2} dx^5 dx^6 = 1$ .

We introduce a constant background for the field strength  $\langle F_{mn} \rangle = f\epsilon_{mn}$ , with  $f$  a quantized number, as typically required in a string model. We split the gauge potential  $A$  into a fluctuation term  $\mathcal{A}$  and a background term  $\langle A \rangle$ , such that  $\langle F \rangle = d\langle A \rangle$ . The background  $\langle A \rangle$  cannot be globally defined in the internal space. On the overlap of different patches, background gauge transformations with a parameter  $\Lambda_0$  are required:

$$\delta_{\Lambda_0} \langle A \rangle = d\Lambda_0, \quad \delta_{\Lambda_0} \mathcal{A} = 0. \quad (2.6)$$

Given the general gauge transformation formulae

$$\delta_{\Lambda_0} A = d\Lambda_0, \quad \delta_{\Lambda_0} B = -\Lambda_0 F + dC, \quad (2.7)$$

it follows that also  $B$  is not globally defined, since it is not possible to absorb  $-\Lambda_0 F$  in  $dC$ . This is clear since the variation of  $dB$ , which is independent of  $C$ , is in general non-trivial:

$$\delta_{\Lambda_0} dB = -d\Lambda_0 \wedge F. \quad (2.8)$$

The last expression can be rewritten according to

$$\delta_{\Lambda_0} dB = -d\Lambda_0 \wedge (\langle F \rangle + d\mathcal{A}) = -d\Lambda_0 \wedge d\mathcal{A} = d(d\Lambda_0 \wedge \mathcal{A}) = \delta_{\Lambda_0} d(\langle A \rangle \wedge \mathcal{A}), \quad (2.9)$$

which shows that, for a new field  $\mathcal{B} = B - \langle A \rangle \wedge \mathcal{A}$ , the quantity  $d\mathcal{B}$  is globally defined. Moreover, the new 2-form  $\mathcal{B}$  will itself be globally defined provided that

$$\delta_{\Lambda_0} \mathcal{B} = -\Lambda_0 F - d\Lambda_0 \wedge \mathcal{A} + dC = 0. \quad (2.10)$$

The required 1-form  $C = C(\Lambda_0, \mathcal{A}, \langle A \rangle)$  (satisfying  $dC = \Lambda_0 F + d\Lambda_0 \wedge \mathcal{A}$ ) can indeed be explicitly given in the case of constant background flux [33].

In conclusion, all the degrees of freedom of  $B$  are now described by a new field  $\mathcal{B} = B - \langle A \rangle \wedge \mathcal{A}$ , that is globally defined in the internal dimensions, and thus has a standard Kaluza-Klein expansion. The gauge transformations of  $\mathcal{B}$  follow from its definition together with eq. (2.3) and the explicit form of  $C$ . They simplify if we focus on  $d\mathcal{B}$  since  $C$  drops out:

$$\delta d\mathcal{B} = -2d\Lambda \wedge \langle F \rangle - d\Lambda \wedge d\mathcal{A}. \quad (2.11)$$

For 4d gauge transformations  $\Lambda = \Lambda(x^\mu)$ , this can be written as

$$\delta (\partial_\mu \mathcal{B}_{56} + \partial_5 \mathcal{B}_{6\mu} + \partial_6 \mathcal{B}_{\mu 5}) = -2\partial_\mu \Lambda \langle F_{56} \rangle - \partial_\mu \Lambda (\partial_5 \mathcal{A}_6 - \partial_6 \mathcal{A}_5). \quad (2.12)$$

Restricting ourselves to the zero-mode level, any dependence of the internal coordinates drops out and we find

$$\delta \mathcal{B}_{56} = -2\Lambda \langle F_{56} \rangle \quad (2.13)$$

for the  $\mathcal{B}_{56}$  zero mode. Note the factor-of-two difference from the naive expectations that one might have for  $B_{56}$  on the basis of eq. (2.3).<sup>7</sup> They are not justified since  $B$  is not globally defined on the internal space and possesses no standard Kaluza-Klein expansion.

We will be interested in the 4d theory arising from the compactification on a supersymmetric  $T^2/Z_2$  orbifold (see section 4 for details). Hence we disregard all 4d vector multiplets which are eliminated by the orbifold projection, as well as the Wilson line degrees of freedom associated with the 5d U(1) gauge theory. What remains are the 4d supergravity and the vector multiplet with gauge field  $A_\mu$  together with three chiral multiplets, the moduli of the compactification. The latter contain the degrees of freedom  $r$ ,  $\phi$ ,  $\tau_1$ ,  $\tau_2$  and two scalars related to the 2-form  $\mathcal{B}$ . The lowest components of the three modulus superfields are [28, 34]

$$S \equiv \frac{1}{2}(s + ic), \quad T \equiv \frac{1}{2}(t + ib), \quad \tau \equiv \frac{1}{2}(\tau_2 + i\tau_1). \quad (2.14)$$

where we have used the definitions

$$t \equiv e^{-\phi} r^2, \quad s \equiv e^\phi r^2 \quad (2.15)$$

and

$$b\epsilon_{mn} \equiv \mathcal{B}_{mn}, \quad \epsilon_{\mu\nu\rho\sigma} \partial^\sigma c \equiv r^4 e^{2\phi} (d\mathcal{B})_{\mu\nu\rho}. \quad (2.16)$$

The Kähler potential, which can be inferred from the kinetic terms for the scalars after dimensional reduction and Weyl rescaling, reads

$$K = -\log(T + \bar{T}) - \log(S + \bar{S}) - \log(\tau + \bar{\tau}). \quad (2.17)$$

Similarly, the gauge-kinetic function is found to be  $h(S) = 2S$  (using the standard conventions of [15]).

Given eq. (2.13), the 4d gauge transformations read

$$\delta b = -2f\Lambda, \quad \delta \mathcal{A}_\mu = \partial_\mu \Lambda, \quad (2.18)$$

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<sup>7</sup>We thank Giovanni Villadoro for discussions about this issue.

which implies that the only nonvanishing component of the Killing vector is  $X^T = -if$ . The resulting  $D$ -term  $D = iK_T X^T = -f/t$  leads to the  $D$ -term potential

$$V_D = \frac{f^2}{2st^2}. \tag{2.19}$$

The same potential also follows directly from the 6d gauge-kinetic term, evaluated in the flux background:

$$\int d^6x \sqrt{g_6} \frac{e^\phi}{4} \langle F_{MN} F^{MN} \rangle = \int d^4x \sqrt{g_4} e^\phi \frac{f^2}{4r^6} \epsilon_{mn} \epsilon^{mn} = \int d^4x \sqrt{g_4} \frac{f^2}{2st^2}. \tag{2.20}$$

This represents a nontrivial check of the fact that the flux is described by the gauging of an isometry from the 4d perspective. (See [35] for a similar computation in heterotic string theory.) Note in particular that, as advertised in the introduction, the gauge transformation acts only on  $T$ , while the gauge kinetic function depends only on  $S$ . Hence, no clash between gaugino condensate and  $D$ -term potential arises. A related situation occurring in the presence of both flux and gaugino condensation on the *same* D7-brane-stack has recently been discussed in the appendix of [11].

### 3. Loop corrections

As an example of a loop correction in the presence of flux, the one-loop Casimir energy of a charged 6d hypermultiplet is computed in this section. This is expected to be the dominant contribution because the constituents of the charged hypermultiplet feel the flux directly. We first derive the Casimir energy for  $T^2$  and then redo the computation with the degrees of freedom that remain in the spectrum for  $T^2/Z_2$ .

The constraints on the gauge and matter content of a consistent anomaly free 6d theory [36] allow the presence of the charged hypermultiplets that we are introducing. Unfortunately, these constraints typically impose also the presence of extra gauge sectors, with extra matter multiplets, whose analysis goes beyond the scope of the present work. In this sense, our model has to be considered as a sector of a complete theory, under the assumption that such a completion does not affect the moduli stabilization studied here.

For the Casimir energy calculation one first has to derive the mass spectra of the charged 6d scalars and Weyl fermions. A 6d hypermultiplet consists of two complex scalars and one 6d Weyl fermion which enter the action in a quite complicated way [2]. We will linearize the  $\sigma$ -model and work with canonical kinetic terms, neglecting the self-interactions of the scalars. This is expected to be a good approximation as long as the mass scale of gauge interactions in 6d is much lower than the 6d Plank scale,  $1/g_{YM,6} \ll M_{Pl,6}$ . Note that the kinetic terms do not contain the 6d dilaton  $\phi$  [2]. In the derivation of the mass spectra we follow [37].

As in the case without flux, the masses of the scalars are given by the eigenvalues of the Laplacian on the compact space. For one minimally coupled complex scalar field with covariant derivative  $\mathcal{D}$ , the Laplacian reads

$$\frac{1}{r^4} (\mathcal{D}_5^2 + \mathcal{D}_6^2), \tag{3.1}$$



where we have used the decomposition of eqs. (2.4) and (2.5), assuming  $\tau_1 = 0$  and  $\tau_2 = 1$ . In the case of a nonzero constant flux the covariant derivatives no longer commute,

$$[\mathcal{D}_5, \mathcal{D}_6] = iF_{56} = if. \tag{3.2}$$

Algebraically, this is equivalent to a one-dimensional harmonic oscillator with unit mass and unit frequency. For positive  $f$  the correspondence is

$$\begin{aligned} \text{Hamiltonian} &\leftrightarrow \frac{1}{2} (\mathcal{D}_5^2 + \mathcal{D}_6^2) \\ \text{position} &\leftrightarrow \mathcal{D}_5 \\ \text{canonical momentum} &\leftrightarrow \mathcal{D}_6 \\ \hbar &\leftrightarrow f. \end{aligned} \tag{3.3}$$

For negative  $f$ , the position and momentum operators have to be interchanged but the mass spectrum is not affected. It reads

$$m_n^2 = \frac{2|f|}{r^4} \left(n + \frac{1}{2}\right), \tag{3.4}$$

where  $n$  is a non-negative integer. Note that the  $n$ -dependence of this mass spectrum is quite different from the usual Kaluza-Klein tower ( $m^2 \sim n_1^2 + n_2^2$ ) resulting from compact dimensions without flux.

Some care has to be taken in deriving the fermionic Kaluza-Klein towers, as is explained in section 14 of [38]. Since the Dirac operator couples righthanded fermions to lefthanded fermions, only its square can have eigenfunctions. The masses of the fermions are determined by

$$m_n^2 r^4 \psi_n = (\Gamma^5 \mathcal{D}_5 + \Gamma^6 \mathcal{D}_6)^2 \psi_n, \tag{3.5}$$

where the  $\psi_n$  are 6d spinors. Observing that

$$(\Gamma^5 \mathcal{D}_5 + \Gamma^6 \mathcal{D}_6)^2 = \mathcal{D}_5^2 + \mathcal{D}_6^2 + i\Gamma^5 \Gamma^6 f, \tag{3.6}$$

it is clear that the problem differs from the bosonic case only by a shift if the spinors are eigenvectors of  $\Gamma^5 \Gamma^6$ . To quantify the effect of the shift, recall that

$$\Gamma^7 = i\gamma^5 \Gamma^5 \Gamma^6, \tag{3.7}$$

and that the 6d chirality is fixed. Decomposing the 6d spinor into a direct sum of two 4d Weyl spinors, we now see that the shift is determined by the chirality of each 4d spinor. The fermionic eigenfunctions are the same as the bosonic ones, the only difference is that they carry an extra chirality index which induces a shift of their masses. The mass spectrum of 4d Weyl fermions reads

$$(m_n^2)_\pm = \frac{2|f|}{r^4} \left(n + \frac{1}{2} \pm \frac{1}{2}\right). \tag{3.8}$$

Another point which has to be addressed is the degeneracy of the spectra. The quickest derivation uses the two-dimensional index theorem, which in our case counts the number of massless fermions. We find that

$$\text{ind}(\Gamma^5 \mathcal{D}_5 + \Gamma^6 \mathcal{D}_6) = \frac{1}{2\pi} \int_{T^2} F = \frac{f}{2\pi} = N. \tag{3.9}$$

Thus the monopole number equals the degeneracy of the state with vanishing mass. It is clear that the ground state of the fermions of opposite chirality has the same degeneracy, because we are considering the same Laplace operator to which merely a constant is added, and thus we find precisely the same eigenfunctions. By the same argument we conclude that the bosonic ground state is  $N$ -fold degenerate.<sup>8</sup> From the oscillator algebra it then follows that all excited states have the same degeneracy as the ground states. Thus every fermionic and every bosonic level is populated by  $N$  states. An extra factor of two arises in the bosonic sector because of the two complex scalars in the hypermultiplet.

With this particle spectrum we directly compute the one-loop effective potential from a four-dimensional perspective. In dimensional regularisation and after Wick rotation to Euclidean space it reads [39]:

$$\sum_{\delta=0, \pm 1/2} (-1)^{2\delta} (2 - 2|\delta|) |N| \sum_{n=0}^{\infty} \int \frac{d^D k}{(2\pi)^d} \ln(k^2 + m_n^2(\delta)), \quad (3.10)$$

where

$$m_n^2(\delta) = \frac{2|f|}{r^4} (n + \frac{1}{2} + \delta) \quad (3.11)$$

are the bosonic ( $\delta = 0$ ) and fermionic ( $\delta = \pm 1/2$ ) mass spectra. This expression is computed in the appendix, giving the result:

$$V_{\text{Casimir}} = \frac{7 |N|^3}{4 (st)^2} \zeta'_R(-2) \cong -0.053 \frac{1}{(2\pi)^3} \frac{|f|^3}{(st)^2} \quad (3.12)$$

Here we have used the quantization condition for the flux, eq. (3.9).

The computation is analogous, albeit technically more involved, in the  $T^2/Z_2$  case. Details are presented in the appendix. The result is:

$$\begin{aligned} V_{\text{Casimir}}^{\pm} &= \frac{7}{4} \frac{1}{(2\pi)^2} \left( \frac{f}{st} \right)^2 \zeta'(-2) J_N^{\pm} \\ &= -0.053 \frac{1}{(2\pi)^2} \left( \frac{f}{st} \right)^2 J_N^{\pm}, \end{aligned} \quad (3.13)$$

where we have defined

$$J_N^{\pm} \equiv |N| \pm 4 \quad (3.14)$$

and the flux quantization condition is

$$\frac{1}{2\pi} \int_{T^2/Z_2} F = \frac{f}{4\pi} = N, \quad (3.15)$$

where the first equality follows from the definition of  $T^2/Z_2$ , the second one from the fact that the flux quantization condition on a sphere is equal to that on a torus.

The two signs in  $V^{\pm}$  stem from the different internal parity that may be assigned to the fermions on the massless level.

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<sup>8</sup>This can also be checked by explicitly computing the zero eigenfunctions. They are given in the appendix.

This correction should be understandable as a correction to the Kähler potential. We found a non-zero Casimir energy because SUSY is broken, which in turn is a result of the flux. The flux was shown to generate a  $D$ -term potential in section 2. We can trace the correction to the  $D$ -term potential back to a correction to the Kähler potential if we assume that the gauge symmetries of our model remain unchanged. Neglecting higher orders in  $1/r$  we find

$$\frac{f^2}{st}(\Delta K)_T = -\frac{1}{(2\pi)^2} \frac{7}{4} \zeta'(-2) \left(\frac{f}{st}\right)^2 J_N^\pm, \quad (3.16)$$

so that we can conclude

$$\Delta K = -\frac{1}{(2\pi)^2} \frac{7}{4} \zeta'(-2) \left(\frac{1}{S+\bar{S}} \log(T+\bar{T})\right) J_N^\pm. \quad (3.17)$$

#### 4. Scherk-Schwarz twists as a source for $W_0$

The presence of closed string fluxes in a type IIB model induces a superpotential  $W_{\text{flux}}(z)$ , that depends on the complex structure moduli  $z$ . The latter are thus stabilized at certain values  $z_{\text{min}}$  and, from the point of view of the low-energy effective theory, the superpotential at the minimum is a constant  $W_0 = W_{\text{flux}}(z_{\text{min}})$ . If  $W_0 \neq 0$ , a SUSY-breaking no-scale model results. In the KKLT construction, a SUSY AdS vacuum is present due to the interplay between  $W_0$  and gaugino condensation. We would like to reproduce this basic structure in our 6d approach. We could in principle introduce a constant  $W_0$  in our model by appealing to the presence of closed string fluxes, since the model can be seen as an intermediate step in the compactification of 10d string theory. In praxis this is not convenient for the following reason. If, on the one hand, closed string fluxes are present in the 6d bulk we consider, we have to start from a more complicated lagrangian. If, on the other hand, the relevant fluxes are present only in the “hidden” four extra dimensions, we loose much of the explicitness of our construction, which is based on a well-known consistent 6d supergravity model. It is therefore convenient to introduce  $W_0$  as the manifestation of Scherk-Schwarz (SS) twists in the two compact extra dimensions as we now discuss in more detail [40] (see [41] specifically for the 6d case).

The 6d supergravity theory studied in section 2 possesses an  $SU(2)_R$  R-symmetry. This can be checked by direct inspection, or by considering it as the result of the compactification of 10d string theory [42]. We follow the second approach. A 10d Majorana-Weyl spinor (a real  $\mathbf{16}$  of  $SO(1,9)$ ) transforms as  $\mathbf{4} \oplus \mathbf{4}'$  under the  $SO(1,5)$  subgroup. The action of the R symmetry group  $SU(2)_R \times SU(2)_{R'}$ , which comes from  $SO(1,9) \supset SO(1,5) \times SO(4) = SO(1,5) \times SU(2)_R \times SU(2)_{R'}$ , is such that the  $\mathbf{4}$  and  $\mathbf{4}'$  transform only under  $SU(2)_R$  and under  $SU(2)_{R'}$  respectively.<sup>9</sup> Consider now the compactification of a 10d  $\mathcal{N} = 1$  model

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<sup>9</sup>Notice that the R-symmetry group does not mix spinors with different 6d chirality. Indeed,  $SO(4)$  can change neither the internal chirality, by definition, nor the 10d chirality, since it is part of  $SO(1,9)$ , and the product of 6d and internal chirality gives precisely the 10d chirality.

A slightly different perspective on the situation can be given as follows: A 10d Weyl spinor (a complex  $\mathbf{16}$ ) transforms under  $SO(1,5) \times SU(2)_R \times SU(2)_{R'}$  as  $(\mathbf{4}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{4}', \mathbf{1}, \mathbf{2})$ . The 10d reality constraint is imposed on each of these two terms independently, without mixing them. This leads to two complex-4-dimensional

on some orbifold limit of  $K3$ , such as  $T^4/Z_n$ . The SUSY generator is a real  $\mathbf{16}$ . Taking the orbifold group to be generated by one of the elements of  $SU(2)_{R'}$ , the supersymmetry associated with the  $\mathbf{4}'$  is broken, while that associated with the  $\mathbf{4}$  is preserved. We thus end up with the R-symmetry group  $SU(2)_R$  since, as explained, the  $\mathbf{4}$  is also a doublet of  $SU(2)_R$ .

In the presence of the  $SU(2)_R$  symmetry, we can compactify the 6d theory on  $T^2$  imposing non-trivial field-identifications. Given a generic  $SU(2)_R$  doublet  $\Phi(x^\mu, x^5, x^6)$  (e.g. the gaugino) we require

$$\Phi(x^\mu, x^5, x^6) = T_5 \Phi(x^\mu, x^5 + 1, x^6), \quad \Phi(x^\mu, x^5, x^6) = T_6 \Phi(x^\mu, x^5, x^6 + 1), \quad (4.1)$$

where the matrices  $T_i$  embed the translations  $t_i$  along the torus coordinate  $x^i$  in the R-symmetry group. Since  $t_5 t_6 = t_6 t_5$ , we also require  $T_5 T_6 = T_6 T_5$ . In case one (or both) of the matrices are non-trivial, we obtain a SS dimensional reduction. If one of the two matrices is trivial, e.g.  $T_6$ , the consistency requirement is automatically satisfied and we can shrink the  $x^6$  direction, obtaining an effective 5d model. From this perspective, the SS twist due to  $T_5$  can be seen as a standard SS twist in a 5d model compactified on  $S^1$ .

For an orbifold compactification of the 6d theory, the rotation operator  $r \in SO(2)$  is also embedded in the R-symmetry group via a matrix  $R$ . A non-trivial embedding is crucial for SUSY not to be broken in a hard way: in case  $R = \mathbb{1}$  the net action of the orbifold on any 4d spinor would indeed result in a non-trivial phase, projecting it out of the spectrum. Having such a non-trivial embedding, extra consistency conditions must be fulfilled, which we now study on a case-by-case basis.

In the case of a  $Z_2$  orbifold,  $r^2 = 1$ ,  $rt_i = t_i^{-1}r$ , and we have to impose these conditions also on the corresponding transformations of the spinors. Non-trivial solutions to these conditions exist [41], as can be easily demonstrated explicitly: The transformation associated with  $r$  is  $\tilde{R} = S(r)R$ , where  $S(r)$  is the phase rotation of the two 4d Weyl spinors coming from a  $\mathbf{4}$  of  $SO(1,5)$ . In the  $Z_2$  case, we have  $S(r) = i\mathbb{1}$ . Choosing  $R = \text{diag}(-i, i)$ , we find  $\tilde{R} = \text{diag}(1, -1)$ .<sup>10</sup> This matrix satisfies the required consistency relations with  $T_i = \exp(i\alpha_i \sigma_2)$ . In case only one of the  $T_i$ 's is non trivial, e.g.  $\alpha_6 = 0$  and  $\alpha_5 = \alpha$ , we can shrink the  $x^6$  direction, obtaining a 5d effective field theory compactified on  $S^1/Z_2$ . In this case it is well known that the continuous SS parameter  $\alpha$  can be described by a tunable constant superpotential  $W_0 \sim \alpha$  [43]. In the rest of the paper, we mainly consider such a  $T^2/Z_2$  compactification, the 4d field content of which was already anticipated in section 2. Notice that with such a field content a constant  $W_0$  leads, in absence of any other effects, to SUSY breaking with zero tree-level potential, as expected in a SS reduction.

In case of a  $Z_3$ ,  $Z_4$  or  $Z_6$  reduction, the field content would be even more appealing, since the  $\tau$  multiplet is projected away. However, the consistency conditions for a SS

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representations of both  $SO(4)$  and  $SU(2)$  which, however, can not anymore be viewed as a  $(\mathbf{4}, \mathbf{2})$ , i.e., as a tensor product of two complex representations.

<sup>10</sup>The computation above can be generalized to the case of the scalars present in a hypermultiplet coming e.g. from a 10d gauge multiplet, which form a doublet of  $SU(2)_R$  and also a doublet of  $SU(2)_{R'}$ . There is no direct action of the rotation on the scalars, which therefore transform only due to the embedding of  $r$  in  $SU(2)_{R \times R'}$  via  $\tilde{R} = R \otimes R'$ . Given  $R$  as above,  $R'$  must be chosen such that  $\tilde{R}^2 = \mathbb{1}$ , e.g.  $R' = R$ .

reduction are now more stringent and cannot be satisfied, not even by discrete SS twists. To see this, let us first give a geometric description of SS breaking on a  $T^2/Z_2$  orbifold. The compact space emerging after the orbifold projection has the topology of a sphere and contains 4 conical singularities, each with an opening angle  $\pi$ . The  $SU(2)_R$  twists create a non-trivial R symmetry holonomy for paths encircling any of the singularities. To avoid hard supersymmetry breaking at the singularities, the size of the corresponding  $SU(2)_R$  rotations has to match the opening angle of the conical singularity. This ensures that, in the local environment of each singularity, a covariantly constant spinor exists. Specifically, using the canonical map from  $SU(2)$  to  $SO(3)$ , the R symmetry twist at each singularity, mapped to  $SO(3)$ , has to be  $\pi$  (matching the rotation in physical space). The overall SUSY breaking to  $\mathcal{N} = 0$  arises from a misalignment of the 4 twists at the 4 conical singularities. This is clearly possible since one can find 4  $SO(3)$  rotations around different axes which altogether give a trivial rotation. (The product of the 4 rotations has to be trivial since a path encircling all 4 singularities can be contracted without encountering another singularity.)

Now consider a  $T^2/Z_3$  orbifold instead. The fundamental space still has the topology of a sphere, but this time with 3 conical singularities, each having an opening angle of  $2\pi/3$ . The R symmetry twist at each singularity (when mapped to  $SO(3)$  in the canonical way) has to be  $2\pi/3$  to avoid hard local SUSY breaking. Given again the global constraint (a loop encircling all 3 singularities is equivalent to a trivial loop), we need to find 3 rotations of magnitude  $2\pi/3$  each which, when multiplied, give  $\mathbb{1}$ . Elementary geometry shows that this is only possible when all 3 rotation axes coincide, in which case an  $\mathcal{N} = 1$  supersymmetry survives in the complete model. Thus, no SS breaking to  $\mathcal{N} = 0$  in 4d is possible. The above argument can be easily extended to the  $Z_4$  and  $Z_6$  cases. In both cases one again has the topology of a sphere with 3 conical singularities. The opening angles are  $(\pi/2, \pi/2, \pi)$  and  $(\pi/3, 2\pi/3, \pi)$  respectively. Three such rotations can not give  $\mathbb{1}$  in total unless their rotation axes coincide, which again leads to  $\mathcal{N} = 1$  in 4d.

Of course, it is also possible to obtain contributions to  $W_0$  by introducing SS twists along some of the 4 hidden extra dimensions of an underlying string model, or by considering localized effects within the  $\mathcal{N} = 1$  sectors at the orbifold singularities (such as brane-localized gaugino condensation).

## 5. Moduli stabilization

In this section we study the stabilization of our model. Besides the  $D$ -term potential induced by the flux and the superpotential generated by the gaugino condensate we assume a constant piece of superpotential which has a negative sign compared to the superpotential from the gaugino condensate. This  $W_0$  is crucial for the stabilization of the modulus  $s$ . We incorporate perturbative corrections in a second step.

To be more precise we start with the following ingredients:

$$K = -\log(T + \bar{T}) - \log(S + \bar{S}) - \log(\tau + \bar{\tau}), \tag{5.1}$$

$$W = \mu^3 \exp(-aS) + W_0, \tag{5.2}$$

assuming for simplicity that  $a$ ,  $\mu$  and  $W_0$  are real. The complete scalar potential is then given by

$$\begin{aligned}
 V &= \frac{1}{st(\tau + \bar{\tau})} \left( \mu^6(a^2s^2 + 2as) \exp(-as) + 2W_0\mu^3as \cos\left(\frac{ac}{2}\right) \exp\left(-\frac{as}{2}\right) \right) + \frac{f^2}{2st^2} \\
 &= \frac{\tilde{V}(s)}{t(\tau + \bar{\tau})} + \frac{f^2}{2st^2},
 \end{aligned}
 \tag{5.3}$$

where the last equation has to be read as a definition of  $\tilde{V}(s)$ . This potential stabilizes both  $s$  and  $t$  at a negative value of  $V$ , as is shown in the following.

Consider first the ‘axionic’ partner of  $s$ , denoted by  $c$ . As  $W_0$  is taken to be negative, while  $a$  and  $\mu^3$  are positive,  $c$  is always stabilized at a value where the cosine is unity. Thus we assume  $c = 0$  in the following. Since the shift symmetry acting on the modulus  $b$  (the ‘axionic’ partner of  $t$ ) is gauged,  $b$  is absorbed in the massive vector boson. Further effects have to be taken into account to stabilize the complex structure modulus  $\tau$ , for which we assume  $2\tau = 1$  from now on. As explained in section 4, the problem of  $\tau$  stabilization does not arise in a  $T^2/Z_n$  ( $n > 2$ ) model, where  $\tau$  is projected away. The only caveat in these cases is that a non-zero superpotential has to be introduced either by SS twists along some of the 4 hidden extra dimensions of an underlying string model, or by localized effects associated with the  $\mathcal{N} = 1$  sectors at the orbifold singularities (such as brane-localized gaugino condensation). Alternatively,  $\tau$  stabilization could result from the non-trivial  $\tau$  dependence of the Casimir energy, which, for simplicity, we do not consider in our computation (see e.g. [44]).

To get some intuition for the stabilization of  $s$  and  $t$ , it is advantageous to first set  $f = 0$  and  $t = 1$ . Then the remaining modulus  $s$  enters the potential in exactly the same fashion as in the KKLT model. At the minimum of the potential,  $s$  has to solve  $D_s W = 0$ , so that we find

$$W_0 + \mu^3 e^{-\frac{as}{2}} (1 + as) = 0.
 \tag{5.4}$$

This is equivalent to minimizing  $\tilde{V}(s)$ . For small  $W_0$  we find the approximate solution

$$as_0 \sim 2 \ln(-\mu^3/W_0).
 \tag{5.5}$$

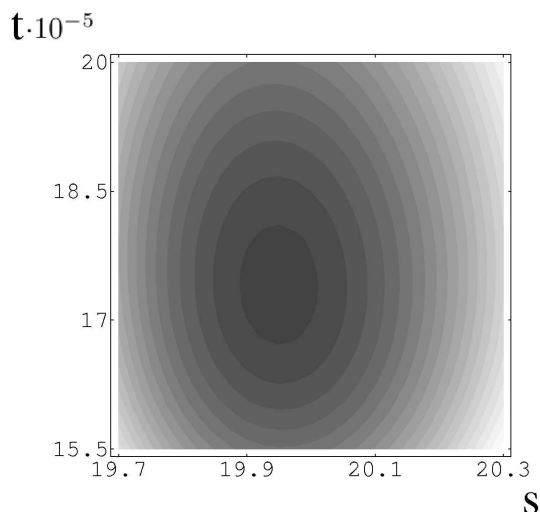
This equation shows that  $as_0$  can be made parametrically large by tuning  $W_0$  to have small negative values. As an example consider  $W_0 = -0.01$ ,  $\mu^3 = 10$  and  $a = 1$ . The result is  $s_0 \sim 20$ .

The approximate value at which  $t$  is stabilized can be found by setting  $s = s_0$ . This is reasonable as the extra  $1/s$  contribution coming from the  $D$ -term potential will not alter the value of  $s$  at the minimum significantly. The resulting potential for  $t$  is then

$$V(t) = \frac{f^2}{2s_0 t^2} + \frac{\tilde{V}(s_0)}{t},
 \tag{5.6}$$

which is minimized by

$$t_0 = -\frac{f^2}{s_0 \tilde{V}(s_0)}.
 \tag{5.7}$$



**Figure 1:**  $F$ -term and  $D$ -term scalar potential as a function of  $s$  and  $t$ .

Equation (5.3) implies  $\tilde{V}(s_0) \sim -10^{-5}$ . In our example we take the flux to have its minimal nonzero value. Due to the quantization condition in the orbifold case,  $f = 4\pi N$ , this is  $4\pi$ . With these numbers eq. (5.7) gives  $t_0 \sim 10^6$ . The exact potential is displayed as a contour plot in figure 1. At the minimum both  $s$  and  $t$  take roughly the expected values. It is worth noting that the minimum of the potential is always at a negative value in this setup, as is best seen from eq. (5.6). The positive piece quadratic in  $1/t$  is dominant for small  $t$ , whereas the negative piece linear in  $1/t$  is dominant for large  $t$ . This tells us that  $V(t)$  comes from positive values and approaches zero from below for  $t \rightarrow \infty$ . So clearly  $V$  is negative in the minimum. This behavior is a result of the simple  $t$  dependence of the Kähler potential.

We now want to comment on the overall consistency of our solution. For the effective 4d description to be valid we need the compactification scale to be below the 6d Plank scale. At the same time the Yang-Mills scale has to be below the 6d Plank scale, but above the compactification scale. We thus require the scales of our model to fulfill  $M_{\text{P},6}^2 > M_{\text{YM},6}^2 > M_{\text{C}}^2$ . The squared Yang-Mills scale in 6d is given by the prefactor of the 6d gauge-kinetic term, so it is equal to  $\exp(\phi_0)$ . With  $\exp(\phi) = \sqrt{s/t}$  we find  $M_{\text{YM},6}^2 \sim 10^{-2}$ , so that the first inequality holds.<sup>11</sup> The compactification scale is set by the volume of the internal dimensions,  $M_{\text{C}}^2 = r_0^{-2} = (s_0 t_0)^{-1/2} \sim 10^{-4}$ , so that the second inequality also holds.

The perturbative corrections of section 3 do not alter the stabilization qualitatively. As a contribution to the effective action, they can simply be added to the scalar potential, which now reads

$$V(t) = \frac{f^2}{2st^2} + \frac{\tilde{V}(s)}{t} - 0.053 \frac{1}{(2\pi)^2} \left(\frac{f}{st}\right)^2 J_N^\pm. \tag{5.8}$$

<sup>11</sup>Note that we have chosen units in which  $M_{\text{P},6} = 1$ .

We see that the minimum is driven to slightly larger values of  $s$  and  $t$ . It is interesting that the loop correction becomes more important than the  $D$ -term potential for large fluxes.<sup>12</sup> This can be understood physically since the degeneracy of the spectrum grows with the flux. Increasing the monopole number, and thus the flux, is equivalent to increasing the degrees of freedom that are present on each Kaluza-Klein level.

## 6. Conclusions

We have approached the set of problems associated with moduli-stabilization and  $D$ -term uplift from the perspective of a simple field-theoretic model. Motivated by the apparent inconsistency between the flux and the gaugino condensate, we have studied an explicit compactification of 6d supergravity, which allows for these two ingredients. This model is directly relevant from a string-theoretic perspective since it can be seen as an intermediate step in the compactification of 10d string theory on a “highly anisotropic” background, with 4 small and 2 large internal dimensions [45].

The gauging and the  $D$ -term potential that arise upon introduction of the flux are determined and found to match in the standard supergravity fashion, confirming that the flux really triggers a  $D$ -term. The modulus that enters the superpotential generated by gaugino condensation is different from the modulus on which the gauged shift symmetry acts. Any potential inconsistency is thus avoided in a natural and attractive way.

To stabilize our model, we discuss two sources for extra potential terms: an R symmetry twist and perturbative corrections. The R symmetry twist is described in terms of a constant superpotential  $W_0$ , so that one of the two main compactification moduli is fixed in a fashion similar to the KKLT model. The other modulus is stabilized by the interplay between the  $D$ -term and the  $F$ -term potential. This mechanism always leads to a non-supersymmetric AdS vacuum in which supersymmetry is broken by both the  $D$ -term and the  $F$ -term.

As a perturbative correction, we considered the Casimir energy of a charged hypermultiplet in the presence of flux. We explicitly calculate these loop corrections for both the  $T^2$  and  $T^2/Z_2$  geometry. From the supergravity perspective, they can be viewed as Kähler corrections, which we also display explicitly. In many cases, our corrections will be more important than the vacuum energy induced by the Scherk-Schwarz twist, since the latter becomes parametrically small in the limit of small  $W_0$ . By contrast, the flux-induced corrections can not be tuned to be small because of flux quantization. It would be interesting to find the counterpart of string-theoretic  $\alpha'$  corrections in our 6d framework and to compare them to the flux-induced Casimir energy.

The above perturbative corrections do not destabilize the non-SUSY AdS vacuum we found previously. However, they are also unable to provide the desired uplift. Thus, a phenomenologically relevant construction would have to include further effects, such as an  $F$  term potential arising in the  $\mathcal{N} = 1$  sectors localized at the orbifold fixed points.

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<sup>12</sup>Recall that  $J_N^\pm \simeq |N| \sim |f|/4\pi$  for large  $|N|$ .



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## A. Computation of the Casimir energy

To compute the Casimir energy we use that

$$\int \frac{d^D k}{(2\pi)^D} \ln(k^2 + m^2) = -\frac{\Gamma(-D/2)}{(4\pi)^{D/2}} m^D. \quad (\text{A.1})$$

We need to compute expressions of the form

$$\sum_n \int \frac{d^D k}{(2\pi)^D} \ln(k^2 + m_n^2(\delta)) = -\frac{\Gamma(-D/2)}{(4\pi)^{D/2}} \sum_n m_n^D(\delta) \equiv I_\delta, \quad (\text{A.2})$$

where

$$m_n^2(\delta) = \frac{2|f|}{r^4} \left(n + \frac{1}{2} + \delta\right) \quad (\text{A.3})$$

are the bosonic/fermionic spectra as computed in section 2. Using the Hurwitz and Riemann zeta functions, denoted by  $\zeta_H$  and  $\zeta_R$  respectively [46], we find that

$$I_\delta = -\frac{\Gamma(-D/2)}{(4\pi)^{D/2}} \left(\frac{2|f|}{r^4}\right)^{D/2} \zeta_H(-D/2, \delta + \frac{1}{2}). \quad (\text{A.4})$$

The limit  $D \rightarrow 4$  for  $\delta = \pm 1/2$  and  $\delta = 0$ , which are the cases of interest to us, can be computed by noting that  $\zeta_R(-2) = 0$  and using the expansions

$$\Gamma(\epsilon - 2) = \frac{1}{2\epsilon} + \mathcal{O}(1) \quad (\text{A.5})$$

$$\zeta_H(\epsilon - 2, 1/2) = (2^{\epsilon-2} - 1)\zeta_R(\epsilon - 2) = -\frac{3}{4}\zeta_R'(-2)\epsilon + \mathcal{O}(\epsilon^2). \quad (\text{A.6})$$

We find that

$$I_0 = \frac{3}{8} \frac{1}{(4\pi)^2} \left(\frac{2f}{r^4}\right)^2 \zeta_R'(-2) \quad (\text{A.7})$$

$$I_{1/2} = I_{-1/2} = -\frac{1}{2} \frac{1}{(4\pi)^2} \left(\frac{2f}{r^4}\right)^2 \zeta_R'(-2), \quad (\text{A.8})$$

where  $\zeta_R'(-2) = -\zeta_R(3)/(4\pi^2) = -0.0304$ . The equality in the last line follows from  $\zeta_H(x, 1) = \zeta_H(x, 0) \equiv \zeta_R(x)$ .

**The  $T^2$  case.** Taking the degeneracy of the spectra and the flux quantization ( $f = 2\pi N$ ) into account, the Casimir energy of one charged hypermultiplet on  $T^2$  can be expressed as

$$\begin{aligned} V_{\text{Casimir}} &= 2|N|I_0 - 2|N|I_{1/2} \\ &= \frac{7|N|^3}{4(st)^2} \zeta_R'(-2) \cong -0.053 \frac{1}{(2\pi)^3} \frac{|f|^3}{(st)^2} \end{aligned} \quad (\text{A.9})$$

**The  $T^2/Z_2$  case.** To find which states are projected away in the orbifold case we need to determine the parity of the zero eigenfunctions. Up to normalization they read

$$\Phi_j = \sum_{m=-\infty}^{\infty} \exp\left(-\frac{1}{2}|f|\left(x_5 - \frac{1}{|2N|}(|2N|m + j)\right)^2\right) \exp(2\pi i(|2N|m + j)x_6), \quad (\text{A.10})$$

where we have used an appropriate gauge [37] and imposed the  $T^2/Z_2$  flux quantization condition  $f = 4\pi N$ . By shifting  $m$  it is easy to see that  $\Phi_j = \Phi_{j+|2N|}$ , so that there are  $|2N|$  distinct eigenfunctions. We find that the parity operation (i.e. the  $Z_2$  rotation) maps  $\Phi_j$  to  $\Phi_{-j}$  and hence to  $\Phi_{|2N|-j}$ . Thus we conclude that

$$\Phi_j^e \equiv \Phi_j + \Phi_{|2N|-j} \quad (\text{A.11})$$

has even parity and

$$\Phi_j^o \equiv \Phi_j - \Phi_{|2N|-j} \quad (\text{A.12})$$

has odd parity. Note that there is no  $\Phi_{|N|}^o$ , but just a  $\Phi_{|N|}^e = 2\Phi_{|N|}$ . Furthermore  $\Phi_{|2N|}$  always has even parity. Besides these exceptions the rest of the spectrum pairs up according to the equations above. The number of even eigenfunctions ( $N_e$ ) is then

$$N_e = |N| + 1. \quad (\text{A.13})$$

To find the number of remaining states on the excited levels, we use the fact that the raising and lowering operators are linear in  $\mathcal{D}_5$  and  $\mathcal{D}_6$ , so that they anticommute with the generator of the  $Z_2$ .

The two complex bosons have different internal parity assignments, so that we find  $|2N|$  of them on each mass level. This is not true for the fermions, because the ground states of different chirality, and hence different internal parity, have different masses:

$$(m_n^2)_{\pm} = \frac{2|f|}{r^4} \left(n + \frac{1}{2} \pm \frac{1}{2}\right). \quad (\text{A.14})$$

We first analyse the tower containing massless states and assume that its fermions have positive internal parity.<sup>13</sup> On the ground state we find  $N_e$  massless fermions that remain in the spectrum. By acting  $2n$  times with the raising operator we find  $N_e$  surviving states on the level  $2n$ , so that we generate a spectrum with masses

$$m_n^2 = \frac{2|f|}{r^4}(2n) \quad (\text{A.15})$$

and degeneracy  $N_e$ . If we consider the  $|2N| - N_e = |N| - 1$  states that are projected away from the ground state and act once with the raising operator, we find  $|N| - 1$  fermions on the first excited level that are even under the orbifold projection.<sup>14</sup> From there we

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<sup>13</sup>If the massless fermions have negative internal parity, the computation is the same with  $N_e$  and  $(|2N| - N_e)$  interchanged.

<sup>14</sup>Note that the raising operators do not change the chirality.

can again act  $2n$  times with the raising operator to find more states that remain in the spectrum. We thus generate a second tower with masses

$$m_n^2 = \frac{2|f|}{r^4}(2n + 1) \tag{A.16}$$

and degeneracy  $|N| - 1$ . We now turn to the fermions of the opposite chirality and hence opposite internal parity. On the ground state of this tower we find  $|2N| - N_e = |N| - 1$  remaining states with masses  $2|f|/r^4$ . By the same argument as above this yields a spectrum

$$m_n^2 = \frac{2|f|}{r^4}(2n + 1), \tag{A.17}$$

with degeneracy  $|N| - 1$ . Acting with the raising operator once on the  $N_e$  ground states that are projected away we find  $N_e$  states on the first excited level that remain in the spectrum. These generate a tower of masses

$$m_n^2 = \frac{2|f|}{r^4}(2n + 2) \tag{A.18}$$

with degeneracy  $N_e$ . As expected, the degeneracy of each state is roughly half of what we found before performing the  $Z_2$  projection.

By appealing to the definitions made at the beginning of the appendix we find

$$V_{\text{bosons}} = |2N|I_0, \tag{A.19}$$

and

$$V_{\text{fermions}}^+ = -8 \left( (|N| + 1)I_{-1/2} + (|N| - 1)I_0 \right) \tag{A.20}$$

if the massless fermions have positive internal parity. If the massless fermions have negative internal parity the fermionic contribution to the Casimir energy reads:

$$V_{\text{fermions}}^- = -8 \left( (|N| - 1)I_{-1/2} + (|N| + 1)I_0 \right). \tag{A.21}$$

Putting everything together and using the explicit expression for  $N_e$  the complete Casimir energy reads

$$V_{\text{Casimir}}^\pm = 7 \left( \frac{N}{st} \right)^2 \zeta'(-2) J_N^\pm, \tag{A.22}$$

where we have defined

$$J_N^\pm \equiv |N| \pm 4. \tag{A.23}$$

The different signs in  $J_N^\pm$  are related to the different parities of the massless fermions: if the parity is positive, the sign is '+', if the parity is negative, the sign is '-'. Note that we recover the Casimir energy of the untruncated spectrum if we add  $V^+$  and  $V^-$  and remember that  $N = f/4\pi$ .

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